On Egg Sandwiches and Neutrinos

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1 Introduction

> "A neutrino is not a big thing to be hit by.
>
> "In fact it's hard to think of anything much smaller by which one could reasonably hope to be hit. And it's not as if being hit by neutrinos was in itself a particularly unusual event for something the size of the Earth. Far from it. It would be an unusual nanosecond in which the Earth was not hit by several billion passing neutinos.
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> "It all depends on what you mean by 'hit' of course, seeing as matter consists almost entirely of nothing at all. The chances of a neutrino actually hitting something as it travels through all this howling emptiness are roughly comparable to that of dropping a ball bearing at random from a cruising 747 and hitting, say, and egg sandwich."
>
> Douglas Adams, *Mostly Harmless*

Adams proposes that the probability of dropping the ball bearing and hitting the sandwich is comparable to the probability of a neutrino interacting as it flies through the Earth. Below is a quick order-of-magnitude assesment of this prediction, based on loose estimates. Surpising results are detailed below.

These computations were made in good faith, without any fudging or manipulation of estimates to achieve a desired result. This calculation is by no means exhaustive, but remains open to further attempts at refinement.

2 Neutrinos and the Earth

Not all neutrinos that pass through the earth actually hit anything. What we wish to measure is the probability of the neutrino interacting with something in the earth assuming that it passes through the planet. One of the dominant modes of interaction with matter is neutrino-electron scattering. (Other modes will enhance our interaction rates by a factor of only 2 or 3, so we'll ignore them.) This interaction is a neutrino bouncing off an electron. We measure this interaction as a *cross section*, that is, what an electron looks like to an incoming neutrino.

To visualize this, imagine shooting an arrow at a distant apple. The chance of hitting the apple is proportional to the size of the apple's silloutte: it's cross section.

The cross section of an electron to neutrinos created in proton-proton reactions in the sun (the most common variety found near earth) is, on average, $11.6 \times 10^{-46}\mathrm{cm}^2$.[1]

Next, we need to know how many of these electrons there are. The earth is electrically neutral, so there is 1 electron per 1 proton. Protons, on average, make up half the weight of the earth (the other half being neutrons). Thus, the number of electrons is

$$\text{Mass of the Earth} \times \frac{\text{One Nucleon}}{\text{Mass of a Nucleon}} \times \frac{\text{One Electron}}{\text{2 Nucleons}} =$$

$$= 1.98 \times 10^{24} kg \times \frac{1\,Nucleon}{1.67 \times 10^{-27} kg} \times \frac{1\,Electron}{2\,Nucleons}$$

$$= 5.9 \times 10^{50} electrons$$

So, the total cross section of all the electrons in the earth is

$$11.6 \times 10^{-46} cm^2/electron \times 5.9 \times 10^{50} electrons = 6.88 \times 10^5 cm^2$$

Let's convert this to meters squared for convience:

$$6.88 \times 10^5 cm^2 \times \frac{1m}{100cm} \times \frac{1m}{100cm} = 68.8m^2$$

So, to neutrinos, the earth looks about the size of a wall seven meters by ten meters! However, the 'bricks' in this wall are scattered over an area the size of the earth. If we look at the earth from space, we see a disk that has an area

$$\pi r^2 = 3.14 \times \left(6.3 \times 10^6 m\right)^2 = 1.25 \times 10^{14} m^2$$

If a neutrino goes wizzing through this area, it has a chance of hitting something that is

$$\frac{Area\ of\ Target}{Total\ Area} = \frac{68.8m^2}{1.25 \times 10^{14} m^2} = 5.5 \times 10^{-13}$$

This is the probability of a given neutrino hitting something as it travels through "all this howling emptiness". The probability of an observable interaction is much less than this. As a brief side note, solar neutrino experiments are able to see actual results because the sun throws off truely incredible numbers of neutrinos. Even at our distance from the sun, there are ˜10^8neutrinos per second passing through an area of 1 meter square. Thus, the rate of neutrino hits occouring in the earth is roughly 7 billion interactions every second.

[1] as taken from **Neutrino Astrophysics**, J. Bachall, Cambridge University Press (1989), pp. 220

3 Egg Sandwiches

Dropping a ball bearing off a 747 and hitting an egg sandwich is not a common occourance. Physics has not devoted a lot of effort to measuring endevours of this sort, but we can make some educated guesses. We know how big a sandwich is.. say 15 cm x 15 cm, but how many of them are there to hit?

Well, let's say that 1 billion people on earth routinely eat egg sandwiches, which counts a good part of the western world. Let's further guess that each of these people eats an egg sandwich once every sixty days. (We may be wrong by a factor of ten either way here. That is, perhaps only 100 million people eat them, or maybe they eat them every 6 days, or whatever. Still, we have a ballpark number to work fro.) The number of sandwiches made every day, on average is:

$$1\,000\,000\,000\,people \times \frac{1\,sandwich}{60\,days} = 1.67 \times 10^7 sandwiches/day$$

Now, these sandwiches are not all outside. Let's say that for 3 months of the year is good picnic weather, and that everyone eats outside one day every three weeks in good weather.

$$1.67 \times 10^7 sandwiches/day \times \frac{3\,months}{12\,months} \times \frac{1\,day}{21\,days} = 1.94 \times 10^5 sandwiches/day$$

This new number is the number of sandwiches that are brought outside every day. They don't last forever, though; let's say we eat our sandwich in half an hour, on average. So, the average number of sandwiches that are outside at any given moment, on average, is:

$$1.94 \times 10^5 sandwiches/day \times 0.5 hours \times \frac{1\,day}{24\,hours} = 4.13 \times 10^3 sandwiches$$

So, there are about four thousand sandwiches exposed to the sky, on average. (This number is probably a bit high; people don't picnic that much, and egg sandwiches aren't that popular.)

Now, each of these four thousand sandwiches measures $15cm \times 15cm = 225cm^2 = 0.0225m^2$, so the total sandwich-area-coverage is $4.13 \times 10^3 sandwiches \times 0.0225m^2/sandwich = 93m^2$.

Finally, the possible area in which we could drop the ball bearing is the size of the surface of the earth, or

$$\frac{4}{3}\pi r^2 = \frac{4}{3} \times 3.14 \times \left(6.3 \times 10^6 m\right)^2 = 1.66 \times 10^{14} m^2$$

and the net probablility of the ball bearing hitting the egg sandwich is thus

$$\frac{Sandwich\,Area}{Area\,of\,Earth} = \frac{93\,m^2}{1.66 \times 10^{14}\,m^2} = 5.6 \times 10^{-13}$$

This calculation implicitly assumes that the 747 in question could be flying anywhere on earth. This is not strictly a good assumption, since we know that 747 flights tend to be flying between cities mainly in North America and Europe. Coincidentally, these same areas also tend, for cultural reasons, to be the largest consumers of egg sandwiches. This correlation may actually increase the eg-bb (egg sandwich - ball bearing) interaction rate. This effect, however can be ignored to first order, and may help mitigate the optimism earlier in choosing egg consumption rates.

4 Conclusions

These to calculations have resulted in very similar answers, differing from each other by only 2%. The author finds this precision spooky. Again, no attempt was made to fudge these results by *a priori* choice of estimates.

Therefore, to within two orders of magnitude, the probability of a neturino performing an electric cross-scattering interaction with an electrion in the earth for a neutrino incident to the earth is approximately equal to the probability of dropping a ball bearing onto a random place on the earth's surface and hitting an egg sandwich.

Adam's hypothetical guess regarding the cross-section of eg-bb interactions is in fact very good. Whether Adams performed this calculation or not is not known, as he provides no numerical evidence for his postulate. His choice of metaphor, however, is uncontestably superb.